FUNCTIONS, STATISTICS, AND TRIGONOMETRY (FST)
Grades 11, 12

Unit of Credit: 1 Year

Prerequisite: Algebra 2 or Consent of Instructor

Course Overview:
Fourth year mathematics courses offered at MCPS include Functions, Statistics, and Trigonometry (FST); Discrete Math; Pre-Calculus; AP Statistics; and AP Calculus.

The “college and career ready” line has been based on evidence from a number of sources, including international benchmarking, surveys of postsecondary faculty and employers, review of state standards, and expert opinion. Students meeting these standards through the first three years of high school mathematics should be well-prepared for introductory mathematics courses in 2- and 4- year colleges. Still, there are persuasive reasons for students to continue on to take a fourth mathematics course in high school.

Research consistently finds that taking mathematics above the Algebra 2 level highly corresponds to many measures of student success. In his groundbreaking report Answers in the Toolbox, Clifford Adelman found that the strongest predictor of postsecondary success is the highest level of mathematics completed (Executive Summary). ACT has found that taking more mathematics courses correlates with greater success on their college entrance examination. Of students taking (Algebra 1, Geometry, Algebra 2, and no other high school mathematics courses), only thirteen percent met the benchmark for readiness for college algebra. One additional mathematics course greatly increased the likelihood that a student would reach that benchmark, and three-fourths of students taking Calculus met the benchmark (ACTb 13).

High school students should be encouraged to select from a range of high quality mathematics options. STEM (Science, Technology, Engineer, Mathematics)-intending students should be strongly encouraged to take Pre-Calculus and Calculus (and perhaps a computer science course). A student interested in psychology may benefit greatly by taking FST followed by AP Statistics. A student interested in starting a business after high school could use knowledge and skills gleaned from a course on mathematical decision-making. Mathematically-inclined students can, at this level, double up on courses—a student taking college calculus and college statistics would be well-prepared for almost any post-secondary career.

Taken together, there is compelling rationale for urging students to continue their mathematical education throughout high school, allowing students several rich options once they have demonstrated mastery of core content.

Standards marked with a (+) may appear either in courses required for all students, or in later courses. In particular, the (+) standards can form the starting point for fourth year courses.

Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appear throughout the high school standards indicated by a star symbol (*). The star symbol sometimes appears on the heading for a group of standards; in that case, it should be understood to apply to all standards in that group. Modeling is best interpreted not as a collection of isolated topics but in relation to other standards.

Number and Quantity Content Standards

Domain: The Complex Number System N-CN

Cluster: Perform arithmetic operations with complex numbers.

3. (+) Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers.
   - I can write the conjugate of a complex number.
   - I can use conjugates to find moduli and quotients of complex numbers.

Cluster: Represent complex numbers and their operations on the complex plane.

4. (+) Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number.
   - I can represent complex numbers on the complex plane in rectangular and polar form (including real and an represent a complex number and its conjugate on the complex plane.
   - I can raise a complex number to an integer power.

6. (+) Calculate the distance between numbers in the complex plane as the modulus of the difference, and the midpoint of a segment as the average of the numbers at its endpoints.
   - I can calculate the distance between numbers in the complex plane as the modulus of the difference.
   - I can calculate the midpoint of a segment in the complex plane.

Domain: Vector Quantities and Matrices N-VM

Cluster: Represent and model with vector quantities.

1. (+) Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g., ***v***, |***v***|, ||***v***||, *v*).
   - I can identify the magnitude and direction of a vector.
   - I can use the appropriate symbols.
   - I can represent a vector in the coordinate plane.

2. (+) Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point.
   - I can calculate the components (horizontal and vertical) using the initial point and the terminal point.

3. (+) Solve problems from a variety of contexts (e.g., science, history, and culture), including those of Montana American Indians, involving velocity and other quantities that can be represented by vectors.
   a. I can solve problems where quantities are represented by vectors.

Cluster: Perform operations on vectors.

4. (+) Add and subtract vectors.
   a. Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes.
      - I can add vectors end-to-end, component-wise, and by the parallelogram rule.
   b. Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum.
      - I can calculate the magnitude and direction of the sum of two vectors.
   c. Understand vector subtraction $\boldsymbol{v} - \boldsymbol{w}$ as $\boldsymbol{v} + (-\boldsymbol{w})$, where $-\boldsymbol{w}$ is the additive inverse of $\boldsymbol{w}$, with the same magnitude as $\boldsymbol{w}$ and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise.
      - I can calculate vector subtraction using the additive inverse.
      - I can graphically represent vector subtraction.
5. (+) Multiply a vector by a scalar.
   a. Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise, e.g., as $c(v_x, v_y) = (cv_x, cv_y)$.
      - I can graphically represent scalar multiplication of vectors.
   b. Compute the magnitude of a scalar multiple $c\boldsymbol{v}$ using $||c\boldsymbol{v}|| = |c|\boldsymbol{v}$. Compute the direction of $c\boldsymbol{v}$ knowing that when $|c|\boldsymbol{v} \neq 0$, the direction of $c\boldsymbol{v}$ is either along $\boldsymbol{v}$ (for $c > 0$) or against $\boldsymbol{v}$ (for $c < 0$).
      - I can calculate the magnitude of a scalar multiple.

Cluster: Perform operations on matrices and use matrices in applications.

6. (+) Use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network.
   - I can represent data using matrices.

7. (+) Multiply matrices by scalars to produce new matrices, e.g., as when all of the payoffs in a game are doubled.
   - I can multiply a matrix by a scalar.

8. (+) Add, subtract, and multiply matrices of appropriate dimensions.
   - I can determine if two matrices can be added or subtracted (same dimensions).
   - I can determine if two matrices can be multiplied (same inner dimensions).
   - I can add/subtract/multiply two matrices.

9. (+) Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties.
   - I can demonstrate that multiplication of matrices is not commutative.

10. (+) Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse.
    - I can add the zero matrix or identity matrix.
    - I can calculate the determinant of a square matrix and if it is non-zero, then that matrix has an inverse.

11. (+) Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors.
    - I can multiply a vector (with one column) by a matrix.

12. (+) Work with 2 × 2 matrices as a transformation of the plane, and interpret the absolute value of the determinant in terms of area.
    - I can determine the area of a parallelogram by taking the absolute value of the matrix formed by the component vectors of two consecutive vertices.

Algebra Content Standards

<u>Domain: Reasoning with Equations and Inequalities A-REI</u>

Cluster: Solve systems of equations.

8. (+) Represent a system of linear equations as a single matrix equation in a vector variable.
    - I can write a matrix equation to represent a system of linear equations.

9. (+) Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimension 3 × 3 or greater).
    - I can use matrices to solve a system of linear equations.

Functions Content Standards

<u>Domain: Interpreting Functions F-IF</u>

Cluster: Analyze functions using different representations.

7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.⋆
    d. (+) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.
    - I can graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.

<u>Domain: Building Functions F-BF</u>

Cluster: Build a function that models a relationship between two quantities.

1. Write a function that describes a relationship between two quantities.⋆
    c. (+) Compose functions. *For example, if T(y) is the temperature in the atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time, then*

T(h(t)) is the temperature at the location of the weather balloon as a function of time.

- I can apply composite functions to real life situations.

Cluster: Build new functions from existing functions.

4. Find inverse functions.

b. (+) Verify by composition that one function is the inverse of another.

- I can verify that a composition of one function is the inverse of another.

c. (+) Read values of an inverse function from a graph or a table, given that the function has an inverse.

- I can determine the inverse values from a graph or a table.

d. (+) Produce an invertible function from a non-invertible function by restricting the domain.

- I can determine the inverse of a function that is not one-to-one by restricting the domain.

5. (+) Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.

- I can solve problems using logarithms and exponents.

Domain: Trigonometric Functions F-TF

Cluster: Extend the domain of trigonometric functions using the unit circle.

3. (+) Use special triangles to determine geometrically the values of sine, cosine, tangent for π/3, π/4 and π/6, and use the unit circle to express the values of sine, cosines, and tangent for x, $\pi + x$, and $2\pi - x$ in terms of their values for x, where x is any real number.

- I can use special triangles to construct the unit circle.
- I can use the unit circle to express the values of sine, cosines, and tangent for x, $\pi + x$, and $2\pi - x$ in terms of their values for x, where x is any real number.

4. (+) Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.

- I can use the unit circle to identify symmetry (odd and even) and periodicity of trigonometric functions.

Cluster: Model periodic phenomena with trigonometric functions.

6. (+) Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.

- I can graph the inverse of a trigonometric function.

7. (+) Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.★

- I can use trigonometric functions to model real life situations.
- I can use inverse trigonometric functions to solve equations.

Cluster: Prove and apply trigonometric identities.

9. (+) Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.

- I can prove the addition and subtraction formulas for sine, cosine, and tangent.

- I can use the addition and subtraction formulas for sine, cosine, and tangent to solve problems.

Modeling Content Standards

Modeling links classroom mathematics and statistics to everyday life, work, and decision-making. Modeling is the process of choosing and using appropriate mathematics and statistics to analyze empirical situations, to understand them better, and to improve decisions. Quantities and their relationships in physical, economic, public policy, social, and everyday situations can be modeled using mathematical and statistical methods. When making mathematical models, technology is valuable for varying assumptions, exploring consequences, and comparing predictions with data.

Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appear throughout the high school standards.

Geometry Content Standards

Domain: Expressing Geometric Properties with Equations G-GPE

Cluster: Translate between the geometric description and the equation for a conic section.

3. (+) Derive the equations of ellipses and hyperbolas given the foci and directrices.
    - I can determine the equation of an ellipse given the foci and the directrices.
    - I can determine the equation of a hyperbola given the foci and directrices.

Cluster: Explain volume formulas and use them to solve problems.

2. (+) Give an informal argument using Cavalieri's principle for the formulas for the volume of a sphere and other solid figures.
    - I can use Cavalieri's principle to derive the volume formula for a sphere.

Statistics and Probability Content Standards

Domain: Using Probability to Make Decisions S-MD

Cluster: Calculate expected values and use them to solve problems.

1. (+) Define a random variable for a quantity of interest by assigning a numerical value to each event in a sample space; graph the corresponding probability distribution using the same graphical displays as for data distributions.
    - I can define a random variable.
    - I can construct a probability distribution table.
    - I can graph a probability distribution table.

2. (+) Calculate the expected value of a random variable; interpret it as the mean of the probability distribution.

- I can calculate and interpret in context the expected value of a discrete random variable.

3. (+) Develop a probability distribution for a random variable defined for a sample space in which theoretical probabilities can be calculated; find the expected value. *For example, find the theoretical probability distribution for the number of correct answers obtained by guessing on all five questions of a multiple-choice test where each question has four choices, and find the expected grade under various grading schemes.*
   - I can develop a theoretical probability distribution and calculate the expected value.

4. (+) Develop a probability distribution for a random variable defined for a sample space in which probabilities are assigned empirically; find the expected value. *For example, find a current data distribution on the number of TV sets per household in the United States, and calculate the expected number of sets per household. How many TV sets would you expect to find in 100 randomly selected households?*
   - I can develop an empirical probability distribution and calculate the expected value.

Cluster: Use probability to evaluate outcomes of decisions.

5. (+) Weigh the possible outcomes of a decision by assigning probabilities to payoff values and finding expected values.
   a. Find the expected payoff for a game of chance. *For example, find the expected winnings from a state lottery ticket or a game at a fast-food restaurant.*
   - I can set up a probability distribution for a random variable representing the pay-off values in a game of chance.

   a. Evaluate and compare strategies on the basis of expected values. *For example, compare a high-deductible versus a low-deductible automobile insurance policy using various, but reasonable, chances of having a minor or a major accident.*
   - I can compare strategies using expected values with respect to decision making.